

Completed records and conditional gauge transport in SMM/DTT

Mohamed Haj Yousef

Abstract

What relational information must an observable record preserve when future tests can coherently compare different histories? We formulate completion as predictive sufficiency for a declared test family. Phase-sensitive interference then forbids identifying histories with distinguishable unitary transports, even when their endpoint structures agree. Independently unframed endpoint carriers cannot supply a natural nonzero comparator from their separate structural data alone; faithful norm- and tensor-preserving comparison therefore requires relational transport obeying the endpoint gauge law. A smooth path correspondence supplies a connection, without selecting its curvature or dynamics. We prove that channels fixing full local carrier algebras also preserve their joint correlations, so local selector forgetting cannot erase a prepared shared reference. An explicit finite channel implements selector completion and retains these correlations. A determinant reference can be transferred covariantly from a coherent reservoir, but cannot be created from invariant preparations by covariant processing. For stipulated rank-three and rank-two carriers, retention of precisely their parallel product-volume tensor selects $S(U(3) \times U(2))$. Coherent invariant amplitude sewing selects the Gauss sector; density-matrix twirling does not. A self-contained appendix derives the carriers only under additional associative source, minimal quadrature, marked-cut, and scalar-extension rules, and exhibits the freedom in spatial dimension. These conditional results distinguish predictive retention, reference resources, gauge geometry, and physical field dynamics; they do not establish an unconditional Standard Model derivation or empirical identification of SMM/DTT.

Keywords: completed records; predictive sufficiency; quantum reference frames; gauge transport; determinant tensor; foundations of physics.

1 The operational question and the claim boundary

A completed record is meaningful only relative to the future experiments for which it is sufficient. If a later interference experiment can distinguish two histories with identical endpoint descriptions, identifying those histories loses predictive information. This provides a concrete question for the Single Monad Model (SMM) and Duality of Time Theory (DTT): which relational distinctions must survive the proposed passage from generative histories to observable records?

The paper answers a restricted mathematical question before considering a temporal interpretation. Its main results are an exclusion criterion for endpoint-only completion, an obstruction to structural-data-only comparison, preservation of joint references under

local record-preserving channels, and a covariant determinant-reference preparation with an explicit resource requirement. These statements concern declared tests, algebras, and channels. Their validity does not require SMM/DTT. The temporal interpretation motivates how these ingredients might occur together; it does not make that realization unique.

The published background has three distinct scopes. Reference [2] organizes behavioral completion, real commutants, and phase-balanced composition. Reference [3] treats phase selection by an oriented real flow on its nonzero-frequency sector. Reference [4] supplies a dual-time representation of specified convolution-memory dynamics. We restate and prove the finite-dimensional results needed here. None of these references establishes the present $3 + 2$ record interface, selected determinant tensor, or autonomous gauge-field Hamiltonian.

The comparison with antecedents concerns hypotheses rather than terminology. Smooth transport and connections are related by established transport-functor geometry [18]. Quantum reference-frame descriptions already distinguish relational resources and frame presentations [1]. Conditional expectations and completely positive semigroups implement the specified selector forgetting [16, 17]. The contribution here is to locate what a sufficient completion must retain, which references such channels cannot erase, and which additional physical resources select the stated frame group and neutral sewing rule. We do not claim historical priority for the general transport correspondence, multiplicative-domain method, or stabilizer construction.

What is established and what is supplied. Theorems concern a stated future-test family, selected scalar phase, declared retained algebras, independent endpoint presentations, and specified comparison and completion maps. The $3 + 2$ carriers, product-volume reference, smooth base, amplitude-sewing rule, and any dynamics require further assumptions. No spatial dimension, gauge coupling, CAR algebra, physical Weyl chirality, family multiplicity, Lorentzian continuum, or empirical realization of SMM/DTT is derived.

The companion mathematical article, *Exact holonomy response and controlled dynamics of singlet-bound lattice loops*, treats specified loop Hamiltonians independently of these operational premises. No result from that separate manuscript is required for a proof here. This division prevents an exact calculation for a selected Hamiltonian from being used as evidence that temporal ontology has selected that Hamiltonian.

2 Behavioral completion and future interference

The words *inner* and *outer* designate generative and completed-observable roles. They do not introduce an additional Lorentzian time coordinate. A generative history is a composable process fragment acting on a state or carrying an amplitude between boundary records. A completed record is a selected output sufficient for a specified family of future tests. Gauge geometry denotes norm- and tensor-preserving transport between retained fibers; gauge dynamics denotes an additional Hamiltonian or action for physically variable transport and its conjugate variables.

Table 1: Independent interfaces and their mathematical conclusions.

Declared input	Conclusion	Independent limitation
Coherent future tests and sufficient records	Observable transport distinctions must survive	Restricting tests changes the quotient
Independent endpoint structures and faithful comparison	Relational links obey the endpoint gauge law	A common frame is additional record data
Smooth local path correspondence	Compatible connection	Curvature and field dynamics are not selected
Full local algebra preservation	Joint carrier correlations survive	Local forgetting does not remove shared references
3 + 2 carriers and parallel product volume	$S(U(3) \times U(2))$ stabilizer	Tensor preparation and global descent are inputs
Neutral invariant amplitude sewing	Gauss sector	Density-matrix twirling is insufficient
Source and completion rules of Appendix A	A seven-state clock and conditional 3 + 2 interface	Quadrature, cut, and retention rules are supplied

2.1 Behavioral completion and the direction of implication

Let a monoid H act on a set X , let $Q : X \rightarrow R$ be a completion map, and let $\mathcal{O}$ be record tests. A test has the form $x \mapsto o(Q(hx))$, with h in a specified class of admissible future histories. Define

$$x \sim y \iff o(Q(hx)) = o(Q(hy)) \text{ for every admissible } (h, o). \quad (1)$$

The tests can be probabilities rather than deterministic outcomes. Equality then means equality of the complete predicted probability distributions.

Proposition 2.1 (Predictively sufficient completion). *The relation (1) is the coarsest equivalence relation preserving all declared future tests. A proposed record map $q : X \rightarrow Y$ is sufficient for them only if $q(x) = q(y)$ implies $x \sim y$. If the future-history class is closed under right composition by an update k , then $x \sim y$ implies $kx \sim ky$, and that update descends to the behavioral quotient.*

Proof. Each test defines a kernel equivalence relation. Their intersection is an equivalence relation and contains every equivalence preserving all the tests. Sufficiency of q requires every test to factor through it, which gives the stated implication. Finally a test of kx is a test of x with future history hk ; closure makes it one of the tests already included. $\square$

This elementary result is the completion principle of [2] in the form needed below. It does not select Q , the tests, or a Hilbert space. Its force is an exclusion rule: a record that discards a distinction detectable by an admitted future experiment is not complete.

Proposition 2.2 (A group quotient cannot erase its descended inverse). *Suppose a group acts on X and every group element descends through a surjective map q . Every descended transformation is invertible. Consequently noninjectivity of q alone does not produce irreversible quotient dynamics from an action whose inverses also descend.*

Proof. The descended actions satisfy $\bar{h}\bar{h}^{-1} = \bar{e} = \text{id}$ and $\bar{h}^{-1}\bar{h} = \text{id}$. They are mutual inverses. $\square$

Thus irreversible completion must restrict the admissible updates to a semigroup, fail to preserve inverse descent, discard an environment, or introduce an operational projection. This qualification is essential when using the quotient language: one must not infer irreversibility merely from a many-to-one observation map. For example, the shift on a two-sided sequence is invertible, while forgetting its past allows the forward one-sided shift to descend and prevents its inverse from descending.

2.2 Future interference forces retention of relational history

The no-shared-frame premise can be complemented by a stronger question: *which history information is completion allowed to forget?* The behavioral criterion of Proposition 2.1 answers this without supposing that all possible shared references are erased.

Let E_x, E_y be finite-dimensional Hermitian carriers. Let $h_1, h_2 : y \rightarrow x$ be two histories with identical endpoint record structures, and let $U_1, U_2 : E_y \rightarrow E_x$ be their unitary transports. Assume a coherent comparison test can prepare $\psi \in E_y$, place the two histories in separate control paths, and recombine them. Its plus-port probability is

$$p_+(\psi, \phi) = \frac{1}{2} [1 + \text{Re}(e^{i\phi} \langle \psi, U_2^* U_1 \psi \rangle)] . \quad (2)$$

The control paths and their relative phase are declared physical reference data. They are not produced by a frame relabeling.

Theorem 2.3 (Endpoint-only completion fails for distinguishable transport). *Suppose the admitted future tests include the coherent comparisons (2), with arbitrary normalized input and a calibrated reference history. If $U_1 \neq U_2$, a predictively sufficient record cannot identify the two histories solely because their endpoint structural records agree. The record must retain their transport distinction or relational data sufficient to reconstruct all its admitted interference statistics. In local frames this retained information transforms by the endpoint gauge-link law.*

Proof. The unitary $W = U_2^* U_1$ differs from I . It therefore has an eigenvector ψ with eigenvalue $e^{i\theta} \neq 1$. At $\phi = 0$ the comparison probability is $(1 + \cos \theta)/2 < 1$, whereas the comparison of U_2 with itself gives one. Hence a declared future test separates the histories, and Proposition 2.1 forbids identifying them in a sufficient record. For independent changes of endpoint frame, $U'_i = g_x^{-1} U_i g_y$, so $W' = g_y^{-1} W g_y$ and the probability is unchanged when $\psi' = g_y^{-1} \psi$. $\square$

This is a necessity of retained relational information, not a necessity of storing a literal microscopic trajectory. An endpoint record enlarged by its transport class is sufficient for these tests. A prepared shared frame can supply that class and must then be counted as part of the record. Conversely, if coherent path references are excluded, tests may identify a central phase or a larger class of transports. The operational test family determines the quotient; a universal ability to resolve every phase is not assumed silently.

Corollary 2.4 (A conditional route from histories to a connection). *Assume retained comparison histories compose, invert under path reversal, are invariant under reparametrization and thin homotopies, and depend smoothly and locally on paths. If their action on retained carriers is faithful, complex-linear, and tensor preserving, they supply smooth gauge transport. Their short-link limit is a compatible connection, and a distinguishable loop gives nontrivial holonomy.*

Proof. Composition and reversal give a path transport functor. In a local trivialization its short-link derivative defines A_μ as in Theorem 5.2; subdivision and smoothness recover its path-ordered transport. Thin homotopies remove changes that sweep out no area. The gauge law follows from the independent endpoint frames. $\square$

The general transport-functor/connection correspondence is established geometry [18]. The application here places the necessity one step earlier: future-test sufficiency disallows deleting observable history transport. A single nontrivial loop can still be a flat topological holonomy. Local curvature requires a nontrivial area-order response of arbitrarily small contractible loops.

3 Selected phase and composite carriers

3.1 Real phase selection, with its zero-mode boundary

Let $U(s)$ be a real orthogonal representation on a finite-dimensional real Hilbert space $V_{\mathbb{R}}$. Its real commutant is

$$\mathcal{A}_{\mathbb{R}} = \{T \in \text{End}_{\mathbb{R}} V_{\mathbb{R}} : TU(s) = U(s)T \ \forall s\}.$$

A compatible phase is $J \in \mathcal{A}_{\mathbb{R}}$ with $J^* = -J$ and $J^2 = -I$. On the complexification, precisely the operators commuting with its canonical conjugation are complexifications of real operators. This identifies the real commutant with the conjugation-fixed part of the complex commutant. Compatible phases are therefore an intrinsic set of skew-adjoint square roots of $-I$, before any phase is selected [2, 3].

Theorem 3.1 (Polar phase of an oriented real flow). *Let $A^* = -A$ generate $U(t) = e^{tA}$. On $V_{\text{dyn}} = (\ker A)^\perp$, put $B = (-A^2)^{1/2}$. Then*

$$J = AB^{-1}, \quad J^2 = -I, \quad J^* = -J, \quad A = JB = BJ.$$

It is the unique orthogonal complex structure in the factorization $A = J|A|$. Reversing A reverses J . A zero mode has no phase selected by this construction.

Proof. A commutes with B by spectral calculus and B is positive and invertible on the stated sector. Hence $J^2 = A^2 B^{-2} = -I$ and $J^* = -B^{-1}A = -J$. The factorization determines J uniquely there. At $A = 0$ it gives no information about a square root of $-I$. $\square$

The published flow theorem also handles unbounded skew-adjoint generators and continuous spectrum at zero, with a bounded polar phase on the orthogonal complement of the kernel [3]. The finite proof above suffices for the clock and auxiliary models in this paper. The positive-frequency construction has substantial antecedents in real quantum

mechanics [9, 10, 11]; its role here is to fix the order in which real dynamics, phase, and complex transport enter the reconstruction.

The selected J makes $(a + ib)v = av + bJv$. A real orthogonal comparator $T : V_y \rightarrow V_x$ is complex-linear exactly when $TJ_y = J_xT$. Thus complex gauge comparison becomes meaningful after phase selection. That does not require a compact inner circle: many nonperiodic real flows select a J as well.

Proposition 3.2 (Phase does not select an energy scale or spectrum). *For a fixed orthogonal complex structure J , every positive invertible self-adjoint B commuting with J gives a skew-adjoint generator $A = JB$ with the same polar phase J . Conversely every generator with this polar phase has that form. Neither J nor its compatible unitary group fixes B , a spectral gap, a hopping strength, or a gauge coupling.*

Proof. $(JB)^* = -JB$, $-(JB)^2 = B^2$, and hence $|JB| = B$. The converse is the polar factorization. Changing B leaves J unchanged and changes its positive frequencies arbitrarily within the commuting class. $\square$

3.2 Composite systems require phase balance

If $V_{\mathbb{R}}$ and $W_{\mathbb{R}}$ have phases J_V, J_W , their raw real tensor product carries two generally different phase actions. Ordinary complex composition requires the balancing relation

$$(J_V v) \otimes w = v \otimes (J_W w).$$

Equivalently retain the range of the orthogonal projector

$$P_{\text{bal}} = \frac{1}{2}(I - J_V \otimes J_W). \quad (3)$$

Indeed $(J_V \otimes J_W)^2 = I$, and on its -1 eigenspace the two phase actions agree. If the complex dimensions are m, n , this range has real dimension $2mn$, the real dimension of $V_{\mathbb{C}} \otimes_{\mathbb{C}} W_{\mathbb{C}}$. Multiplying the projected elementary real tensor by $\sqrt{2}$ gives the usual product-norm convention. All composite Hilbert spaces below use this selected complex tensor product. This is a composition assumption, not a consequence of the one-particle polar theorem; network tests can depend on it [3, 12].

4 Selector completion and joint references

4.1 Why a copied selector is not enough

No-go theorem 4.1 (Projector-erasure obstruction). *Let $\mathcal{E} : \mathcal{A} \rightarrow \mathcal{B}$ be a conditional expectation that fixes $\mathcal{B}$ pointwise. If a microscopic line projector p belongs to $\mathcal{B}$, then $\mathcal{E}(p) = p$ and it is not forgotten. Therefore one cannot erase the projectors that select a multiplet while preserving the full algebra containing those same projectors pointwise.*

One explicit resolution is standard form: selectors and carriers are commuting left and right actions of the same matrix algebra.

Definition 4.2 (Canonical standard-form record channel). For $V \cong \mathbb{C}^m$, identify

$$L^2(M_m, \text{Tr}) \cong V_L \otimes \bar{V}_R.$$

Let M_m act on the left by L_a and its commutant act on the right by R_b . For $\gamma_m > 0$ and $t \geq 0$, define

$$\mathcal{E}_m(X) = \frac{I_L}{m} \otimes \text{Tr}_L(X), \quad \Phi_t^{(m)} = \mathcal{E}_m + e^{-\gamma_m t}(\text{id} - \mathcal{E}_m).$$

Theorem 4.3 (Exact selector forgetting with carrier preservation). For $m = 3, 2$, $\gamma_m > 0$, and each $t > 0$:

$$\begin{aligned} \text{Fix}(\Phi_t^{(m)}) &= I_L \otimes M_m(\mathbb{C})_R, \\ \mathcal{E}_m(p_a \otimes I_R) &= \frac{I_L}{m} \otimes I_R, \\ \mathcal{E}_m(I_L \otimes b) &= I_L \otimes b. \end{aligned}$$

At $t = 0$, $\Phi_0^{(m)} = \text{id}$ and its fixed algebra is the full algebra. The right algebra is the common fixed algebra of the semigroup and the fixed algebra of every positive-time map. The convergence estimate is exact for all $t \geq 0$:

$$\|\Phi_t^{(m)} - \mathcal{E}_m\|_{2 \rightarrow 2} = e^{-\gamma_m t}.$$

After positive 3–2 block dephasing, the full fixed algebra is

$$M_3(\mathbb{C})_R \oplus M_2(\mathbb{C})_R.$$

Proof. $\mathcal{E}_m$ is the Hilbert–Schmidt orthogonal projection onto the right commutant, hence is unital, trace preserving, completely positive, and idempotent. The displayed semigroup follows from $\mathcal{E}_m^2 = \mathcal{E}_m$. Its fixed space is exactly the range of $\mathcal{E}_m$. Block dephasing removes off-diagonal intertwiners. $\square$

Corollary 4.4 (No designated retained in/out polarization). *The two designated left defect projectors have the same record image. Therefore the left selector does not supply a preferred polarization of the right carrier. The full right matrix algebra nevertheless contains rank-one projectors and self-adjoint involutions: these observables are not erased. The right carrier has frame group $U(2)$ only if no additional retained tensor, Hamiltonian constraint, or record coupling singles out a right polarization.*

Remark 4.5 (Passive frames versus physical transformations). A passive frame change transforms states, observables, and generators together; it is not a transformation of the state alone while every observable remains fixed. Erasing a left selector does not identify all distinct right states or make every right unitary physically trivial. The record channel is compatible with right frame changes, but local frame redundancy is a separate representational principle. Also, Φ_t is primitive on the selector factor, not on the entire standard-form algebra, whose right fixed algebra is nontrivial.

Theorem 4.6 (Reversible inner steps cannot complete an exact record). *Let $m \geq 2$ and let $\mathcal{E}_m$ be the standard-form selector channel of Definition 4.2, acting on $\mathcal{A}_m = B(V_L \otimes \bar{V}_R)$. If p_a, p_b are distinct orthogonal rank-one projectors on V_L , then*

$$D_{ab} = (p_a - p_b) \otimes I_R \neq 0, \quad \mathcal{E}_m(D_{ab}) = 0.$$

For every reversible step represented on this same algebra by a unitary (or orthogonal, after complexification) conjugation α , including any finite composition of such steps,

$$\|\alpha - \mathcal{E}_m\|_{2 \rightarrow 2} \geq 1.$$

Thus an iteration of reversible inner-clock steps cannot implement exact selector-forgetting completion on this algebra. For the specified mixing semigroup, $\Phi_t^{(m)} - \mathcal{E}_m = e^{-\gamma m t}(\text{id} - \mathcal{E}_m)$, so exact completion occurs only as $t \rightarrow \infty$, or by taking the record map $\mathcal{E}_m$ itself as an operational projection.

Proof. Definition 4.2 sends both $p_a \otimes I_R$ and $p_b \otimes I_R$ to $I_L/m \otimes I_R$. Normalize D_{ab} to unit Hilbert–Schmidt norm. Any reversible conjugation preserves that norm and remains injective, whereas $\mathcal{E}_m(D_{ab}) = 0$. Hence $\|(\alpha - \mathcal{E}_m)(D_{ab})\|_2 = 1$, proving the bound. The semigroup identity follows from $\mathcal{E}_m^2 = \mathcal{E}_m$. $\square$

This is a limited operational necessity for separating inner generation from completed records. It permits a unitary dilation on a larger system with information transferred to an environment, and it does not derive the particular channel $\mathcal{E}_m$, the seven-state clock, or a physical identification of this separation with DTT. At finite t , $\Phi_t^{(m)}$ is invertible as a linear superoperator but is not reversible as a quantum channel: it attenuates D_{ab} , and its inverse is not a positive map on the selector algebra.

Proposition 4.7 (Local record preservation does not erase joint frames). *Let x, y be completed records, with right-carrier algebras $\mathcal{B}_x = B(E_{R_x}) \otimes I$ and $\mathcal{B}_y = I \otimes B(E_{R_y})$ embedded in their joint observable algebra. If a unital completely positive joint channel Ψ fixes $\mathcal{B}_x$ and $\mathcal{B}_y$ pointwise, then it fixes the generated algebra $\mathcal{B}_x \vee \mathcal{B}_y \cong B(E_{R_x} \otimes E_{R_y})$ pointwise. In particular, applying independent instances of $\mathcal{E}_m$ to left selector factors leaves every pre-existing right–right correlation unchanged. Exact local selector erasure therefore does not establish R2 for the joint completed records.*

Proof. For $b \in \mathcal{B}_x$ or $b \in \mathcal{B}_y$, pointwise fixation of its full $*$ -algebra gives $\Psi(b^*b) = b^*b = \Psi(b)^*\Psi(b)$ and likewise for bb^* . Thus b belongs to the multiplicative domain of Ψ . For $b_x \in \mathcal{B}_x$ and $b_y \in \mathcal{B}_y$ it follows that $\Psi(b_x b_y) = \Psi(b_x)\Psi(b_y) = b_x b_y$. Such products span the joint right algebra. $\square$

For example, a prepared right-carrier correlation $|\Omega\rangle\langle\Omega|$, with $|\Omega\rangle = m^{-1/2} \sum_{i=1}^m |i\rangle_x \otimes |i\rangle_y$, is retained by these channels. Its selected pairing of bases is additional joint data; independent endpoint frame automorphisms do not fix that *particular* preparation. This does not invalidate independent passive changes of presentation, nor require that every right-correlated state define a faithful linear comparator. It shows why R2 must specify which *structural tensors and reference preparations* the completion supplies, rather than asserting that local depolarization deletes every possible cross-record reference.

The finite construction in Section 8 implements completion while retaining the explicit reference resources.

5 Independent endpoint presentations and faithful transport

5.1 What follows from SMM/DTT and what is added

The SMM common-source hypothesis motivates a common compositional origin. DTT distinguishes generative inner evolution from completed outer-time records. These state-

ments do not, by themselves, specify which information the record retains or how different records can be compared. We therefore make their proposed operational completion explicit:

- (R1) **Completed-record sufficiency.** Predictions about completed events depend on their record data, not on a selector label erased by the record map. Distinct right carrier states remain distinct physical data.
- (R2) **No unrecorded frame identification.** The structural data at a record are its Hermitian carrier spaces, selected phase algebra, block identities, and any declared retained tensors. They contain neither a named internal basis nor a prescribed common frame relating distinct records. Local presentations may be selected independently. This principle concerns passive coordinate choices, not active equivalence of all carrier states.
- (R3) **Faithful local comparison.** When neighboring records are compared, the comparison is complex-linear, invertible, preserves the carrier norm and retained structural tensors, has an inverse for reversed comparison, and composes along successive records. It must compare arbitrary carrier amplitudes, rather than only one preferred state.
- (R4) **Smooth correspondence limit.** On an emergent smooth record chart, short comparisons approach identity in a chosen smooth local presentation and admit a first-order expansion. Neither smooth spacetime nor its dimension is derived here.

R1 is implemented for erased labels by the channel of Section 4. R2 is consistent with that channel but is not implied by depolarization alone: a shared externally recorded frame would violate it. R3 is an additional correspondence requirement for reversible comparison. R4 is a continuum assumption. Calling these operational core principles defines the strengthened model; it does not retroactively prove that they follow from the bare ontological postulates.

5.2 Local structure does not select a comparator

For one record let $E = E_3 \oplus E_2$ with its Hermitian metric, phase unit, splitting, and total unit volume tensor. Its adapted frames form a torsor for $G_{\text{src}} = S(U(3) \times U(2))$: the group acts freely and transitively on compatible frames, with no distinguished element. Two records have independent torsors until relational data are supplied.

Theorem 5.1 (No structural-data-only faithful comparison). *There is no nonzero block-preserving linear comparator $C_{xy} : E_y \rightarrow E_x$ constructed naturally from only the separate structural data of two records, if their adapted frame automorphisms act independently. In particular no such construction can provide the faithful comparison required by R3.*

Proof. Naturality under endpoint automorphisms, which fix the structural input, requires

$$C_{xy} = g_x^{-1} C_{xy} g_y \quad \text{for all } g_x, g_y \in G_{\text{src}}.$$

Take $g_y = I$. On the triplet choose $g_x = (\zeta I_3, I_2)$ with $\zeta = e^{2\pi i/3}$; it belongs to G_{src} and forces the triplet block to vanish. On the doublet choose $g_x = (I_3, -I_2)$, also in G_{src} ; it forces the doublet block to vanish. Thus $C_{xy} = 0$. The same argument applies to ordinary $U(m)$ carriers whenever their automorphism group has no fixed vector in the defining representation. $\square$

The theorem does not deny a comparator supplied by a common ambient source, an overlap, or a recorded reference frame. Such objects are additional relational data and are precisely what separate endpoint structural data lack. Nor does it say that passive frame freedom makes all active states physically equivalent.

Theorem 5.2 (Gauge transport is the minimal faithful comparison structure). *Under R1–R3 for the retained 3+2 carrier with a global total-volume tensor, a faithful comparison requires additional relational isomorphisms between retained fibers. Their matrices in local adapted frames are G_{src} -valued links C_{xy} , with*

$$C'_{xy} = g_x^{-1} C_{xy} g_y, \quad C_{yx} = C_{xy}^{-1}. \quad (4)$$

Their products compare successive records; closed products are defined up to conjugation. Under R4, short links give a compatible connection with the usual inhomogeneous transformation law. No connection, covariant derivative, or Yang–Mills action is assumed in R1–R3. Here “minimal” means that endpoint data must be supplemented by relational comparison; it is not a theorem of uniqueness or categorical minimality among all possible implementations.

Proof. The preceding theorem excludes a faithful comparator from structural endpoint data alone. A norm-preserving complex-linear isomorphism is unitary by the polarization identity; preservation of the splitting and total tensor makes it a G_{src} isomorphism. Writing its intrinsic action $E_y \rightarrow E_x$ in independently changed frames gives (4). Composition cancels intermediate frames, and a closed product transforms by conjugation at its starting point. Under R4 write

$$C_{x, x+\epsilon e_\mu} = I + \epsilon A_\mu(x) + o(\epsilon).$$

Expanding $g_x^{-1} C_{x, x+\epsilon e_\mu} g_{x+\epsilon e_\mu}$ gives

$$A'_\mu = g^{-1} A_\mu g + g^{-1} \partial_\mu g.$$

Unitarity makes A_μ anti-Hermitian, and tensor preservation imposes the total-trace constraint. The differential comparison is $D_\mu = \partial_\mu + A_\mu$, derived from the link expansion. Its Leibniz rule follows from differentiating scalar multiplication rather than being postulated to define a connection. $\square$

If the global total-volume tensor is not retained, the same necessity argument uses the automorphism group of the actual declared record structure instead of G_{src} . The particular determinant-linked group depends on the additional tensor-retention premise. This establishes necessity of gauge-covariant transport within the operational completion of SMM/DTT. It establishes neither a unique transport law nor independent gauge-field degrees of freedom. Closed-loop formulations encode the same comparison content without introducing local potentials as observable entities. Relational reference-frame changes already have a substantial literature [1]; the application here is to this paper’s explicit completion and record model.

The full carrier modules and reference tensors must remain distinct from the retained observable algebra. Conjugation on $M_3 \oplus M_2$ annihilates scalar block phases, while the defining carrier and phase-sensitive comparisons can resolve them. A record restricted to adjoint observables therefore has a different operational quotient. Even a faithful comparison resource need not provide every required test: if the canonical normalized singlet $\Omega_m = m^{-1/2} \sum_{a=1}^m e_a \otimes \bar{e}_a$ is transported on its defining-carrier factor once around

a prescribed loop and then projected back onto its root singlet, the success probability is $|\text{Tr } W/m|^2$, which equals one for every central holonomy $W = e^{i\theta} I_m$. Coherent comparison against a calibrated alternative history, as in (2), resolves that central phase. The distinguishability claim is relative to that explicit experiment family.

5.3 Which comparisons have observable content?

Proposition 5.3 (Tree reduction and cycle observables). *On a finite connected record graph with inverse links, a choice of frames can make every spanning-tree link identity. The remaining edges give $|\mathcal{E}| - |\mathcal{V}| + 1$ fundamental cycle comparators, defined up to simultaneous conjugation at a root. Thus local link matrices are gauge dependent, while cycle spectra and trace deficits carry frame-independent comparison information.*

Proof. Fix a root frame. For a tree edge directed from parent x to child y , choose $g_y = C_{xy}^{-1} g_x$. Its transformed backward comparator is identity. Recursion defines all frames uniquely once the root is chosen. Every non-tree edge closes one fundamental cycle; changing the root frame conjugates all such products together. $\square$

A tree has no independent loop holonomy, and a one-dimensional smooth chart has zero local curvature. Inner-time compactness may support a global loop but does not force a nonzero spacetime curvature two-form. Curvature records nontrivial local cycles in the smooth comparison geometry; its existence requires more than unobservable labels.

6 Determinant references and their preparation cost

Take stipulated Hermitian carriers E_3 and E_2 with complex ranks three and two. At this stage their structural frame group is $U(3) \times U(2)$. The optional source and cut construction in Appendix A supplies these ranks under additional hypotheses; none of the preceding operational theorems requires that particular construction. Product-volume retention is a separate interface, whose channel and preparation are now explicit.

6.1 An explicit auxiliary phase register

Let $\mathcal{R} = \mathbb{C}^4$ with ordered basis $|00\rangle, |10\rangle, |01\rangle, |11\rangle$. These denote auxiliary volume-label occupations, not physical carrier states. Define

$$Q_{\mathcal{R}} = \text{diag}(0, -6, 6, 0), \quad \mathcal{L}_{\phi}(X) = -\frac{\gamma_{\phi}}{2} [Q_{\mathcal{R}}, [Q_{\mathcal{R}}, X]], \quad \gamma_{\phi} > 0.$$

Here the charges are the determinants of the primitive relative module weights -2 and 3 , multiplied by block ranks three and two.

Theorem 6.1 (Product-volume coherence without carrier decoherence). *$\Xi_t = e^{t\mathcal{L}_{\phi}}$ is a unital trace-preserving completely positive semigroup. For matrix units,*

$$\Xi_t(|a\rangle\langle b|) = e^{-\gamma_{\phi} t (q_a - q_b)^2 / 2} |a\rangle\langle b|.$$

The total-volume coherence $C_{\text{tot}} = |11\rangle\langle 00|$ is fixed, whereas the separate volume coherences $C_3 = |10\rangle\langle 00|$ and $C_2 = |01\rangle\langle 00|$ decay as $e^{-18\gamma_{\phi} t}$. The register fixed algebra is $M_2 \oplus \mathbb{C} \oplus \mathbb{C}$, of complex dimension six.

Proof. The generator equals $\gamma_\phi(QXQ - \{Q^2, X\}/2)$ and is of Lindblad form. Alternatively, Ξ_t is an average of conjugations $e^{i\alpha Q} X e^{-i\alpha Q}$ with centered Gaussian variance $\gamma_\phi t$. Its action on matrix units and fixed charge eigenspaces gives all assertions. $\square$

For each standard-form block take the tensor-product channel $\Phi_t^{(m)} \otimes \Xi_t$; inter-block coherences can be removed by an independent block-dephasing generator. This preserves $I_L \otimes b_R \otimes I_{\mathcal{R}}$ for every physical right observable. The auxiliary fixed algebra is real mathematical data and is not silently deleted from the full channel. Its identification as a label register rather than a propagating matter sector is a modeling assumption. Therefore the carrier fixed algebra alone has dimension $9 + 4 = 13$, while including this register gives $(9 + 4)6 = 78$.

To attach the register to volume tensors, use $|10\rangle$ and $|01\rangle$ as labels for the two block wedges and $|11\rangle$ for their product. A prepared nonzero C_{tot} retains a product-phase reference. The channel preserves but does not create that reference. No right carrier state is averaged over the relative phase. This explicitly separates label forgetting from physical charge decoherence.

Proposition 6.2 (Primitive phase-calibration descent). *On the algebraic exterior space $\Lambda(E_3 \oplus E_2)$ let N_3 and N_2 count triplet and doublet occupancy. This is a tensor construction, without a physical fermionic interpretation here. The unique primitive nonzero block-scalar integer generator annihilating the fully occupied bidegree $(3, 2)$ is, up to sign,*

$$Q_{\text{rel}} = -2N_3 + 3N_2.$$

Its passive phase action on frame tensors multiplies the triplet and doublet defining modules by $e^{-2i\alpha}$ and $e^{3i\alpha}$. The separate top wedges have charges -6 and $+6$, while their product and the vacuum-to-full-completion operator have charge zero. Averaging this phase in an auxiliary frame-label register forgets both separate determinant frames while preserving their product. This averaging is not a channel on physical carrier states: applying it there would generally remove charged coherences and could conflict with preservation of the full right matrix algebra. The channel above realizes the tensor-label interface; its physical identification and prepared reference remain part of the record hypothesis.

Proof. A block-scalar integer generator is $aN_3 + bN_2$. Neutrality of the full bidegree gives $3a + 2b = 0$, hence $(a, b) = k(-2, 3)$ for $k \in \mathbb{Z}$. Primitivity gives $k = \pm 1$. The wedge charges and product neutrality follow by adding the defining weights; the phase-register theorem gives their stated noise behavior. $\square$

Locally let e_3 and e_2 be unit wedges of the two retained carriers. Then $\tau_{\text{src}} = e_3 \otimes e_2$ is a local unit tensor. Global descent of these products is a separate assumption: neither a local marker nor the four-state register determines all transition functions. A trivialization allows the tensor reference to be supplied globally; a local preparation alone does not prove this.

Theorem 6.3 (Canonical total determinant cancellation). *Let τ_{src} be a stipulated globally retained unit tensor in $\det E_3 \otimes \det E_2$, represented locally by the product-volume reference. It survives the declared record channel, although no individual spectral or defect line and no separate block volume is retained as a named tensor. Its unitary stabilizer is*

$$S(U(3) \times U(2)) = \{(u_3, u_2) : \det u_3 \det u_2 = 1\}.$$

Compatibility here includes $\nabla\tau_{\text{src}} = 0$; it is a restriction on allowed transport, not an automatic property of any spectral connection. Consequently total determinant holonomy is trivial on every loop, including torsion and flat-holonomy loops.

Proof. Local orthonormal frames provide nonvanishing local volume tensors. The global assertion assumes that their product patches as the retained tensor specified by the stated total-volume retention rule; local marker-fixed phases alone do not prove this gluing. A product wedge is unchanged by determinant-one internal frame changes but does not retain a named basis. Its declared reference retains the product phase. By Proposition 6.2, the two separate block wedges are charged and averaged away, whereas their product is neutral. A block unitary acts on the product wedge by $\det u_3 \det u_2$, giving the stabilizer. Parallelity of a global unit tensor makes its holonomy exactly one; this is stronger than zero curvature. $\square$

Corollary 6.4 (Global quotient and unique relative Abelian factor). *There is a canonical isomorphism*

$$S(U(3) \times U(2)) \cong \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6},$$

induced by

$$(g_3, g_2, z) \longmapsto (z^{-2}g_3, z^3g_2).$$

The kernel has six elements. Because the selector fixed algebras are scalar and the total determinant removes one of the two block-center phases, exactly one Abelian factor survives.

Proof. The displayed homomorphism has image in the determinant-one product. For any (u_3, u_2) in that product choose z with $z^6 = \det u_2 = (\det u_3)^{-1}$. Then $g_3 = z^2u_3$ and $g_2 = z^{-3}u_2$ have determinant one, proving surjectivity. Its kernel consists of $(z^2I_3, z^{-3}I_2, z)$ with $z^6 = 1$. The Lie algebra of the two block centers has dimension two; the total trace constraint removes one dimension. $\square$

Proposition 6.5 (Smooth compact phase atlas). *Let $(g_{3,ab}, g_{2,ab})$ be source-compatible frame transitions. Then*

$$h_{ab} = \det g_{2,ab} = (\det g_{3,ab})^{-1}$$

is a smooth $U(1)$ Čech cocycle. It defines the relative line

$$L_{\text{rel}} = \det E_2 \cong (\det E_3)^*.$$

No separate phase-atlas hypothesis is needed.

No-go theorem 6.6 (Curvature-only determinant argument). *The local condition $\text{Tr } F_3 + \text{Tr } F_2 = 0$ does not eliminate a nontrivial flat total determinant line. Controlling all periods requires a global trivial-holonomy condition, supplied here by a parallel unit tensor such as τ_{src} .*

Proof. On S^1 let D be a trivial complex line with connection $d + i\alpha d\theta$, where $\alpha \notin \mathbb{Z}$. Its curvature is zero but its holonomy is $e^{-2\pi i\alpha} \neq 1$. Take $E_3 = D \oplus \mathbb{C}^2$ and $E_2 = \mathbb{C}^2$ with the induced direct-sum connection. Then the total determinant has that same flat holonomy despite the vanishing total trace curvature. A parallel global unit tensor is impossible in this example. $\square$

6.2 Preparing a product volume consumes a reference

Pinching preserves a vacuum-to-product coherence but cannot create it from a diagonal register state. An explicit preparation makes that distinction measurable. At one vertex put $D = \det E_3 \otimes \det E_2$ and restrict the register to $\mathbb{C}|0\rangle \oplus D$. Supply a reservoir $S = \mathbb{C}|0_S\rangle \oplus D^*$. The canonical unit tensor in $D \otimes D^*$ defines a unitary V which swaps $|0, 0_S\rangle$ with that tensor and fixes the other two occupation sectors. It commutes with the full $U(3) \times U(2)$ action: both swapped vectors have trivial total character.

Proposition 6.7 (A covariant preparation and its reference cost). *For a chosen unit tensor $\tau \in D$, prepare $|+\tau^*\rangle_S = (|0_S\rangle + |\tau^*\rangle)/\sqrt{2}$. Then*

$$V(|0\rangle \otimes |+\tau^*\rangle_S) = \frac{|0\rangle + |\tau\rangle}{\sqrt{2}} \otimes |\tau^*\rangle_S. \quad (5)$$

The phase coherence has been transferred from the reservoir into the product-volume register. Conversely no covariant channel acting on an invariant initial register and invariant ancillary state can generate a nonzero vacuum-to-product coherence when D has its nontrivial determinant character.

Proof. The swap maps the first initial component to $|\tau, \tau^*\rangle$ and leaves $|0, \tau^*\rangle$ fixed, which proves (5). This definition is independent of the phase used to write the canonical tensor $\tau \otimes \tau^*$. A covariant channel maps invariant inputs to invariant outputs. The off-diagonal operator $|\tau\rangle\langle 0|$ transforms by $\det g_3 \det g_2$, a nontrivial character, so its expectation in an invariant output is zero. Tracing out a covariantly coupled invariant reservoir does not change that conclusion. $\square$

The covariance obstruction and phase-asymmetry monotones are established resource-theory ideas [19, 20]. The next bound specializes them to the determinant-line reference and identifies an attainable resource requirement for the specified preparation.

Proposition 6.8 (A quantitative coherence resource bound). *Let $\mathcal{T}_{\text{in}}$ and $\mathcal{T}_{\text{out}}$ be normalized Haar twirls for the full $U(3) \times U(2)$ action, and let Λ be a covariant trace-preserving completely positive map. For a normalized input state ρ , any standalone output coherence $z = \langle \tau | \Lambda(\rho) | 0 \rangle$ obeys*

$$|z| \leq \frac{1}{2} \|\rho - \mathcal{T}_{\text{in}}(\rho)\|_1. \quad (6)$$

On the output subspace $\mathbb{C}|0\rangle \oplus D$, the distance from an output state to the set of invariant states equals $|z|$ in trace-distance convention. In particular a pure balanced determinant reference requires input asymmetry of at least $1/2$ in (6).

Proof. Covariance gives $\mathcal{T}_{\text{out}}\Lambda = \Lambda\mathcal{T}_{\text{in}}$. Trace-norm contractivity on Hermitian differences therefore implies

$$\|\Lambda(\rho) - \mathcal{T}_{\text{out}}\Lambda(\rho)\|_1 \leq \|\rho - \mathcal{T}_{\text{in}}(\rho)\|_1.$$

The product-determinant character is nontrivial for the full group, so the output twirl deletes z . An off-diagonal self-adjoint witness $e^{-i \arg z} |0\rangle\langle \tau| + e^{i \arg z} |\tau\rangle\langle 0|$ has operator norm one and expectation $2|z|$ in the untwirled difference. Duality of trace and operator norms proves the bound. On the two-line subspace, deleting only z gives a difference with eigenvalues $\pm|z|$, attaining the lower bound for distance to invariant states. A balanced pure reference has $|z| = 1/2$. $\square$

This is a cost in phase asymmetry, not a thermodynamic energy estimate. It also distinguishes the full block group, under which the desired product reference is charged, from the relative label-noise subgroup, under which it is neutral and hence preserved. In the explicit swap preparation the bound is attained by the coherent reservoir.

Equivalently the swap is a calibrated pulse generated on the neutral two-dimensional sector by a Hamiltonian coupling vacuum to the canonical $D \otimes D^*$ tensor. Its pulse area and the coherent reservoir preparation are resources. Starting instead from both vacua, a partial pulse can make a globally invariant superposition of $|0, 0_S\rangle$ and $|\tau, \tau^*\rangle$, but its reduced register has zero vacuum-to-product coherence. Entanglement with a retained dual reference is not a standalone prepared determinant reference.

After the actual preparation (5), the phase channel of Theorem 6.1 preserves the product coherence. The stabilizer of the prepared splitting, metrics, and tensor is $S(U(3) \times U(2))$, as proved earlier. The channel has not generated this tensor from symmetric data, and no target group was required to define the enlarged covariant swap. Nevertheless choosing precisely this tensor resource and correlated relative noise is an additional physical choice. Global patching of tensors at different vertices is also an input; finite local preparation does not prove bundle descent.

7 Invariant sewing and Gauss constraints

A passive gauge-link law does not by itself restrict the state space to gauge singlets. To obtain Gauss constraints we specify a *composition rule*: when regions are joined at an internal record, their boundary amplitudes are sewn independently of an arbitrary internal frame. The precise rule uses coherent averaging, and differs from discarding a classical frame label in a density matrix.

Axiom 7.1 (Invariant internal sewing). *At each internal record v , let R_v be the unitary representation of the retained compact connected frame group G on all incident boundary and matter indices. Sewing replaces a boundary amplitude by its normalized Haar average $P_v\psi = \int_G R_v(g)\psi \, dg$. The initial physical sector has no external charge at such an internal record. Boundary records may carry specified charge representations. Representations at distinct vertices are assumed to commute, as in tensor-product links with commuting left/right endpoint actions and local matter factors.*

This is additional physical content. Independence of the name of a frame motivates it but does not uniquely force coherent averaging. Different charge sectors or reference-carrying boundaries are legitimate alternatives.

Theorem 7.2 (Invariant sewing gives the Gauss sector). *P_v is the orthogonal projector onto G -invariant amplitudes. If $R_v(e^{sT_a}) = e^{is\mathcal{G}_v^a}$, then*

$$P_v\psi = \psi \iff \mathcal{G}_v^a\psi = 0 \text{ for every generator } a,$$

on the common generator domain. Sewing at different vertices gives the intersection of these Gauss sectors. A Hamiltonian commuting with all $R_v(g)$ preserves the sewn subspace.

Proof. Haar invariance gives $R_v(h)P_v = P_v$, $P_v^* = P_v$, and $P_v^2 = P_v$. Every invariant vector is fixed by the integral, proving the range statement. Differentiating group invariance gives the generator conditions. Conversely zero generators imply invariance under their one-parameter subgroups, which generate a connected group. On a link, the actions of

its two endpoints are left and right translations and commute, so the vertex projectors commute. Commutation of a Hamiltonian with the representation implies commutation with each projector. $\square$

On $L^2(G^\mathcal{E})$ the link $U_{wv} : E_v \rightarrow E_w$ transforms actively as $U_{wv} \mapsto h_w U_{wv} h_v^{-1}$. The generator at a vertex is the sum of its signed left/right electric generators plus its matter charge. This is the Hamiltonian Gauss law of lattice gauge theory [6]. Its sign is fixed by this endpoint convention. For $G = S(U(3) \times U(2))$ there are twelve generators per vertex; the two block centers share one relative generator. No thirteenth independent Abelian constraint is introduced.

Proposition 7.3 (Frame twirling is not singlet projection). *The invariant density matrices of $\mathcal{T}(\rho) = \int_G R(g)\rho R(g)^* dg$ need not be supported on $\text{Ran } P$. Thus ordinary ignorance of a frame does not establish the singlet Gauss sector.*

Proof. On a nontrivial irreducible representation of dimension d , the density matrix I_d/d is invariant under twirling, while the Haar average of amplitudes is zero because the representation contains no invariant vector. More generally, for $\mathcal{K} = \bigoplus_\lambda V_\lambda \otimes M_\lambda$, twirling produces blocks of the form $I_{V_\lambda}/\dim V_\lambda \otimes \rho_\lambda$; it does not delete nontrivial λ sectors. $\square$

This distinction identifies exactly what must be derived from a future microscopic completion rule. If it supplies coherent internal sewing and the neutral initial sector, Gauss constraints follow. If it supplies only twirling, one obtains invariant density operators with possible charges. A gauge-invariant Hamiltonian preserves this sector once it is selected; symmetry of the Hamiltonian does not independently choose it.

8 A finite completion instrument with explicit retained memory

A generic finite construction is enough to join the channel and reference arguments. It deliberately does not specify a generated plaquette Hamiltonian. Let $\mathcal{V}$ be a supplied finite set of records and take

$$\mathcal{D} = \bigotimes_{v \in \mathcal{V}} \mathcal{D}_v, \quad \mathcal{D}_v = \bigoplus_{a \in \{3,2\}} (L_{v,a} \otimes E_{v,a}), \quad \dim L_{v,a} = \dim E_{v,a} = a.$$

Define $\mathcal{E}_{\text{rec}}$ by deleting off-diagonal block coherences and applying $\mathcal{E}_a$ on each left selector. Its range is the full joint right algebra

$$\mathcal{B}_R = \bigotimes_{v \in \mathcal{V}} (M_3 \oplus M_2), \quad \dim_{\mathbb{C}} \mathcal{B}_R = 13^{|\mathcal{V}|}.$$

At each vertex attach the four-dimensional volume-label register of Section 6.1. Let $q = \text{diag}(0, -1, 1, 0)$ and define

$$\mathcal{D}_\phi(X) = \frac{1}{3} \sum_{k=0}^2 Z^k X Z^{-k}, \quad Z = e^{2\pi i q/3}. \quad (7)$$

This is the exact charge-eigenspace pinching onto $M_2 \oplus \mathbb{C} \oplus \mathbb{C}$. The equality and opposite signs of the two nonzero *label* charges are a stated noise rule. They are not physical carrier

weights. For arbitrary ranks (r, s) the same construction retains their product-volume line, so this channel does not select $3 + 2$.

Let $\mathcal{K}$ be a finite probe Hilbert space with a projector P , put $Q = I - P$, and choose a self-adjoint probe Hamiltonian H . All probe data and couplings are supplied. With $U_\tau = e^{-i\tau H/\hbar}$, put

$$\mathcal{E} = \mathcal{E}_{\text{rec}} \otimes \mathcal{D}_\phi^{\otimes |\mathcal{V}|} \otimes \mathcal{D}_P, \quad \mathcal{D}_P(X) = PXP + QXQ, \quad \Phi(X) = U_\tau^* \mathcal{E}(X) U_\tau. \quad (8)$$

The probe unitary has identity action on the record and volume-label factors. The entire map is unital, trace preserving, and completely positive.

Theorem 8.1 (Fixed algebra of a finite completion instrument). *For every τ ,*

$$\text{Fix } \Phi = \mathcal{B}_R \otimes (M_2 \oplus \mathbb{C} \oplus \mathbb{C})^{\otimes |\mathcal{V}|} \otimes \{P, U_\tau\}', \quad (9)$$

where the last commutant is in $B(\mathcal{K})$. If $0 < \tau \text{diam Spec } H < 2\pi\hbar$, it equals $\{P, H\}'$. Every local and joint right-carrier observable is fixed pointwise.

Proof. $\mathcal{E}$ is a Hilbert–Schmidt orthogonal projection. If $X = U_\tau^* \mathcal{E}(X) U_\tau$, equality of Hilbert–Schmidt norms implies $\mathcal{E}(X) = X$, and then $[X, U_\tau] = 0$. The converse is immediate. The range of $\mathcal{E}$ is the displayed tensor product with probe factor $\{P\}'$. Intersecting with the commutant of a unitary acting solely on that factor yields (9). The spectral no-alias condition makes $\lambda \mapsto e^{-i\tau\lambda/\hbar}$ injective on $\text{Spec } H$; U_τ and H consequently have the same spectral projections. $\square$

No convergence to this fixed algebra is asserted. For example, probe sectors on which completion is ineffective can retain rotating coherences under U_τ . Fixed observables and all nondecaying information are different notions.

The instrument has probe Kraus operators $M_1 = PU_\tau$, $M_0 = QU_\tau$. For a supplied initial probe vector s , sequential outcome probabilities are

$$\Pr(r_1, \dots, r_n) = \|M_{r_n} \cdots M_{r_1} s\|^2.$$

Fresh preparation after each outcome is a different reset protocol and consumes an additional resource. Frequent probe measurements likewise change the coherent dynamics; a Hamiltonian response calculation cannot automatically be applied to its repeatedly measured limit.

Each completion step admits a finite reversible dilation. Selector depolarization uses a fresh maximally mixed swap register; block and P/Q pinching use controlled phase gates and fresh classical registers; (7) uses a fresh uniform three-state register controlling Z^k . Irreversibility appears when those registers are discarded and excluded from subsequent tests. The finite clock in Appendix A does not supply their preparation or replenishment.

This construction also makes the preparation restriction for independently unframed endpoint structures explicit. Product right states and no cross-record entangled reference are compatible with those structures. Correlated right inputs are equally legitimate states of the larger model, and their correlations survive. A common temporal origin does not imply reference-free preparation. If the probe Hamiltonian instead acts directly on the protected physical right carriers, their pointwise fixation must be recomputed; it does not follow from the theorem.

9 Conditional synthesis and discriminating counter-models

Theorem 9.1 (Operational reconstruction with separated hypotheses). *The following implications hold independently:*

- (i) *A record sufficient for the coherent tests of Theorem 2.3 retains every transport distinction those tests resolve.*
- (ii) *Under R1–R3, faithful comparison of independently unframed carriers requires relational isomorphisms with the endpoint gauge law. R4 and the stated smooth transport hypotheses give the connection limit.*
- (iii) *For stipulated $3 + 2$ carriers, retention of precisely a parallel global product-volume tensor selects $S(U(3) \times U(2))$. Covariant processing cannot create that standalone reference from invariant input resources.*
- (iv) *A unital completely positive map fixing both full endpoint carrier algebras also fixes their joint algebra. The finite channel of Theorem 8.1 realizes selector forgetting while preserving joint references.*
- (v) *Neutral invariant amplitude sewing selects Gauss-invariant amplitudes. Density-matrix twirling alone does not select that sector.*

Proof. The statements are respectively Theorem 2.3; Theorems 5.1 and 5.2 with Corollary 2.4; Theorem 6.3 and Proposition 6.7; Proposition 4.7 and Theorem 8.1; and Theorem 7.2 with Proposition 7.3. $\square$

Each implication is conditional on its own input. A sufficient record does not generate its carriers; the carriers do not select adjacency; passive covariance does not select a force; sewing does not select a Hamiltonian. The optional source construction provides a finite interface, rather than collapsing these distinctions.

9.1 A sharp limit: the bare source permits flat transport

Proposition 9.2 (Flat realization of the bare reconstruction assumptions). *The finite source, phase-compatible defect, selector channel, and retained product tensor admit a realization with constant carrier projectors, identity comparison links, zero curvature, and no autonomous gauge field. Therefore those bare assumptions do not entail interacting Yang–Mills dynamics.*

Proof. Take the exact seven-state source and its scalar-extended defect, identically at every point of a trivial record chart. Use the same standard-form and phase-register channels pointwise, with a fixed total-volume reference. Set $C_{xy} = I$ in that fixed trivialization. All finite source and record identities hold, the connection is flat, and no kinetic gauge field has been introduced. Passive local changes of presentation produce pure-gauge matrices but do not alter the zero curvature. $\square$

The distinction is logically useful: R2–R3 make relational transport necessary, but nonzero curvature and autonomous dynamics require further physical input. Any stronger claimed derivation from the same assumptions would have to exclude this explicit flat realization.

9.2 The scope of a necessity claim

Theorems 5.1 and 5.2 concern the data needed to compare independently unframed records. They do not say that SMM/DTT is required in every microscopic realization of gauge theory. Conventional lattice gauge theory starts from link variables and a specified local gauge symmetry [5, 6]; other many-body mechanisms can produce emergent gauge excitations [8]. These are logically distinct starting points.

A stronger SMM/DTT necessity statement would have to name a class $\mathfrak{C}$ of microscopic update rules and an independently specified target $\mathfrak{T}$: a record network with a three-dimensional spatial scaling regime, relational G_{src} comparisons, and autonomous gauge response. The criteria defining $\mathfrak{C}$ must be motivated without building inner time, a seven-state clock, three spatial axes, the gauge group, or their desired conclusions into its definition. One would then prove that every member of $\mathfrak{C}$ realizing $\mathfrak{T}$ contains an identifiable inner generation/completion distinction and the requisite record quotient, or else produce a counterexample. Such a statement would establish conditional uniqueness within $\mathfrak{C}$, not an unrestricted ontological necessity. In particular, a counterfactual rule that omits completion must be analyzed with the same observables and low-energy criteria as the original rule. A model whose every link is the identity demonstrates insufficiency of the current axioms, but the existence of a flat *configuration* or vacuum does not disqualify a dynamical gauge theory.

9.3 Which stronger claims require new results?

A microscopic derivation must make the test family, record channel, reference preparation, adjacency, and comparison Hamiltonian outputs of a rule chosen independently of the desired target. In particular, a derivation of the determinant-linked group must explain why the product-volume reference is retained while separate volumes and additional shared tensors are not. The preparation theorem locates the required asymmetry resource; it does not select that resource.

The first useful extension would specify an independently motivated family of completion instruments and classify their retained tensor categories and reference requirements. A second would derive a coherent sewing rule rather than only covariance or twirling. A third would relate a generated record adjacency to operational displacements and a controlled continuum. Appendix C supplies countermodels and one sufficient growth criterion against which any dimensional claim can be tested. A gauge-field Hamiltonian and its electric and magnetic parameters require a further microscopic selection and matching calculation.

The causal or quantum-mechanical role of SMM/DTT would become discriminating only after a physical system, admissible preparations, measurable tests, and calibrated parameter map are specified. The present theorem excludes inadequate records within the chosen experiment class. It neither demonstrates that all microscopic realizations require the temporal ontology nor identifies its realization in nature.

10 Conclusion

Future-test sufficiency constrains what completion may erase. Under coherent phase-sensitive comparison, transport distinctions survive; under independent endpoint structures, faithful comparison requires additional relational data with the gauge-link law.

Local record-preserving channels also preserve full joint carrier correlations, making reference resources unavoidable rather than removable by local selector noise. A determinant reference can be transferred from a coherent reservoir, while symmetric preparations cannot create it covariantly. Its stipulated retention selects a specific frame stabilizer, and a separately specified coherent sewing rule selects the neutral Gauss sector.

These results provide precise operational content for a proposed SMM/DTT reconstruction while retaining its logical limits. Selection of the source interface, retained tensors, spatial adjacency, Hamiltonian, and physical matching remains a program of further theorems and discrimination, not a consequence of naming generative and completed time.

A An optional finite source and phase-compatible $3+2$ interface

Axiom A.1 (Compositional source). *Source amplitudes form a finite-dimensional real unital associative algebra. Composition is bilinear and closed, and a positive quadratic intensity is exactly multiplicative.*

Axiom A.2 (Nonclassicality). *Source composition is noncommutative. This excludes the scalar and ordinary complex alternatives before any recurrence order is considered.*

Axiom A.3 (Minimal reversible completion). *Every center-free real source direction is assigned its own faithful conjugate quadrature two-plane. A single reset state closes the orbit. The reset orbit is regular and there are no inaccessible, duplicated, or spectator states.*

Axiom A.4 (Canonical open record and full-event completion). *Microscopic resolution data act in the left standard representation and are mixed primitively; retained carrier data are the right commutant. The completion first forms real cut defects and then extends scalars through the selected phase algebra. A separate auxiliary tensor register retains the neutral product volume coherence and mixes the relative block phase. The physical choice to retain that coherence is an assumption. The record map is not a closed automorphism and does not act as matter-state decoherence.*

Remark A.5 (Physical assumptions and target dependence). The source axioms do not explicitly name a gauge group, but absence of its name is not a proof of target independence. Associativity, one quadrature pair per source direction, a regular reset orbit, the doubled defect interface, the standard-form separation, and retention of only the total block volume are substantive restrictions. In particular, the total-volume retention rule uses the already constructed $3+2$ ranks and specifies a tensor category whose stabilizer is the stated group. Their independent motivation and robustness under alternative record mechanisms are part of the research program.

A.1 Ontological interpretation and mathematical content

SMM motivates a common compositional source rather than independent primitive spatial entities. DTT motivates a distinction between inner reconstruction and completed outer-time records. Neither statement alone selects a division algebra, a seven-state clock, a complex scalar field, a smooth spacetime, or local gauge freedom. The model supplies explicit additional assumptions at each interface. Complex phases here use a selected J

with $J^2 = -I$; they must not be identified with a split-complex temporal unit satisfying $j^2 = +1$.

The distinction between a common origin and a common observable frame is central. A microscopic marker may fix a frame before recording, while the record need not retain that frame. The construction below separates microscopic selector labels from retained state spaces. A local gauge description additionally requires independently chosen charts over a smooth record base; a finite source at a single point cannot itself establish locality.

A.2 Quaternionic source rigidity and order selection

Lemma A.6 (Frobenius source classification). *Under Axioms A.1 and A.2, $A \cong \mathbb{H}$.*

Proof. Positive norm multiplicativity excludes zero divisors. Finite dimensionality and associativity imply that every nonzero element is invertible. Frobenius' theorem leaves only $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$; noncommutativity selects $\mathbb{H}$. $\square$

Corollary A.7 (Intrinsic displacement module). *The center-free module*

$$W = A/Z(A) \cong \text{Im } \mathbb{H}$$

has real dimension three. The norm induces its Euclidean metric. Quaternion multiplication gives the nonzero alternating form

$$\omega(u, v, w) = \langle uv, w \rangle,$$

which fixes an orientation. The connected inner automorphism group is $SO(W)$.

Lemma A.8 (Count under the stipulated independent-quadrature rule). *A nontrivial continuously oriented real frequency mode has dimension at least two. Under Axiom A.3, which assigns an independent such plane to each of the d source directions, the nonmarker clock has dimension exactly $2d$.*

Proof. A continuous orthogonal flow on a real line is trivial. A nonzero real skew-adjoint frequency therefore occurs on a real two-plane, represented by a conjugate pair of complex eigenvalues. The independent-plane assignment and absence of spectators in Axiom A.3 give the stated count. Orthogonal dynamics alone does not require a separate plane for each source direction; that assignment is the substantive additional rule. $\square$

Definition A.9 (Explicit microscopic clock). *For an oriented orthonormal source frame $F = (f_1, \dots, f_d)$, let*

$$\mathcal{H}_F = \mathbb{R}u_0 \oplus W_c \oplus W_s, \quad N = 1 + 2d.$$

Let $c(f_k)$ and $s(f_k)$ be the two copies of f_k . Define

$$\begin{aligned} K_F u_0 &= u_0, \\ K_F c(f_k) &= \cos(2\pi k/N) c(f_k) + \sin(2\pi k/N) s(f_k), \\ K_F s(f_k) &= -\sin(2\pi k/N) c(f_k) + \cos(2\pi k/N) s(f_k), \end{aligned}$$

and

$$\Omega_F = \frac{1}{\sqrt{N}} u_0 + \sqrt{\frac{2}{N}} \sum_{k=1}^d c(f_k).$$

The reset channel is $\mathcal{C}_F(\rho) = |\Omega_F\rangle\langle\Omega_F| \text{Tr } \rho$.

Lemma A.10 (Regularity and characterization of reset vectors). *The vectors $e_n = K_F^n \Omega_F$, $n \in \mathbb{Z}_N$, form an orthonormal basis. Conversely, for the simple spectrum of K_F , a cyclic vector has an orthonormal orbit exactly when every spectral weight equals $1/N$. The explicitly marked vector in Definition A.9 fixes a phase convention; uniform weights alone do not make that vector unique.*

Proof. The inner product $\langle e_m, e_n \rangle$ is the inverse discrete Fourier transform of the uniform spectral measure on all N th roots of unity, hence equals δ_{mn} . Conversely, orthogonality of all orbit translates says that the Fourier transform of the spectral weights is the Kronecker delta, so the weights are uniform. Reality pairs conjugate phases. The displayed marked-vector formula selects the cosine signs and reset sign; source orientation selects between K_F and K_F^{-1} . These are convention choices, not additional consequences of orbit orthogonality. $\square$

Theorem A.11 (Order seven under the stated construction). *The source of Lemma A.6 has $d = 3$. The clock of Definition A.9 therefore has $N = 1 + 2d = 7$, and Lemma A.10 makes its first-return order seven.*

Proof. By Corollary A.7, $d = 3$. Minimal quadrature gives dimension $1 + 2d = 7$. A regular orbit is an orthonormal basis, so its length equals the Hilbert-space dimension. The explicit eigenvalues are all seventh roots, and no proper positive power fixes them all. $\square$

Proposition A.12 (Exclusion of nearby orders). *Within the axioms:*

- (a) $N = 3, 5$ provide only one or two nontrivial conjugate planes and cannot read out all of W ;
- (b) $N = 9, 11, \dots$ contain at least one additional Fourier plane and violate no-spectator minimality;
- (c) even N contain a real -1 character, which is not a conjugate quadrature plane and violates the all-nonzero-modes phase rule.

No-go theorem A.13 (Octonionic fork). *If associativity is removed while positive norm composition is retained, the octonions become admissible. Their center-free imaginary module has rank seven and the same minimal-dilation rule gives order fifteen. This is an alternative model branch. Dropping associativity merely admits it; it does not force an octonionic source or by itself provide an empirical falsifier.*

A.3 Carrier, complex structure, defects, and gaps

Let $Re_n = e_{-n}$. Then $R^2 = 1$, $R\Omega = \Omega$, and $RKR = K^{-1}$.

Lemma A.14 (Odd carrier and physical isotropy). *The reflection-odd subspace has dimension three and is canonically the sine-quadrature copy W_s . Assume the spatial frame-selector preparation is averaged over $SO(W)$. Then the only retained positive bilinear form on this output is a scalar multiple of the quaternionic norm, and the only retained orientation is the quaternionic orientation. Hence the retained output metric has isotropy group $SO(3)$. This is not yet a construction of three-dimensional spacetime.*

Proof. Reflection is $+1$ on the cosine copy and -1 on the sine copy. The declared frame average is $SO(W)$ -invariant, and the defining representation on W is irreducible. Schur's lemma makes every invariant symmetric bilinear form scalar. The source three-form fixes orientation. $\square$

For the complexified regular clock define

$$P_k = \frac{1}{7} \sum_{n=0}^6 e^{-2\pi i k n / 7} K^n, \quad z_k = \sqrt{7} P_k \Omega, \quad k = 1, 2, 3.$$

These are the positive-frequency projectors and marker-calibrated vectors. Complexification here is a device for displaying the spectral lines; the physical complex scalar structure is then identified through the selected polar unit.

Lemma A.15 (Positive-frequency complex structure). *On the nonfixed sector, let $A = \log K$ be the oriented principal skew-adjoint logarithm. Its polar phase*

$$J = A|A|^{-1}$$

is the unique orthogonal complex structure satisfying $A = J|A|$. The positive-frequency lines $\mathbb{C}z_k$, $k = 1, 2, 3$, form $E_3 \cong \mathbb{C}^3$.

Proof. The six nonfixed eigenvalues of K are the three conjugate pairs $e^{\pm 2\pi i k / 7}$, $k = 1, 2, 3$. Their principal oriented logarithms are nonzero, so Theorem 3.1 applies. The positive member of each pair contributes one complex line. The regular marker has nonzero projection onto every such line, and the vectors z_k are normalized by its uniform spectral weights. $\square$

A.3.1 The cut obstruction and a phase-compatible completion functor

Proposition A.16 (Raw marked cuts do not preserve the source phase). *Let J be the polar phase on the nonfixed real clock sector, extended by zero on the fixed mode, and $P_\Omega = I - |\Omega\rangle\langle\Omega|$. Then $[P_\Omega, J] \neq 0$. For the regular seven-state source,*

$$\|[P_\Omega, J]\| = \sqrt{6/7}.$$

Consequently the raw cut cannot be declared complex-linear over the original polar structure.

Proof. The marked vector has weight $1/7$ on the fixed mode and $6/7$ on the nonfixed modes. Put $v = J\Omega$. Skew-adjointness gives $\langle\Omega, v\rangle = 0$, and $\|v\|^2 = 6/7$. Direct calculation gives $[P_\Omega, J] = |\Omega\rangle\langle v| + |v\rangle\langle\Omega|$, whose nonzero eigenvalues are $\pm\|v\|$. $\square$

The obstruction is exact. It cannot be removed by simply renaming the ordinary scalar i as the selected phase J .

Definition A.17 (Scalar-extension completion rule). *On the nonfixed source space define the real algebra*

$$\mathbb{C}_J = \{aI + bJ : a, b \in \mathbb{R}\}, \quad J^2 = -I.$$

This is a copy of the complex field with distinguished oriented imaginary unit J . For every real interface module D , define

$$\mathcal{S}_J(D) = D \otimes_{\mathbb{R}} \mathbb{C}_J.$$

For a real map $L : D \rightarrow D'$ set $\mathcal{S}_J(L) = L \otimes 1$. Scalar multiplication acts on the second factor. The norm is obtained from the real metric and the usual norm $a^2 + b^2$ on the phase algebra.

This is a specified composition rule, not a claim that every microscopic defect is already a complex subspace of the original clock. Its merit is precise compatibility: it uses the already selected phase algebra without requiring $LJ = JL$ on a real domain where such a condition is inappropriate. Under source conjugacy $J' = UJU^{-1}$ the map $aI + bJ \mapsto aI + bJ'$ canonically identifies the scalar algebras; reversal of the selected orientation conjugates them.

Theorem A.18 (Phase-compatible doubled defect). *Let $V_\Omega = \Omega^\perp$ over $\mathbb{R}$ and*

$$T_R = P_\Omega K P_\Omega|_{V_\Omega}, \quad D_R = \begin{pmatrix} 0 & T_R^* \\ T_R & 0 \end{pmatrix} \quad \text{on } V_\Omega \oplus V_\Omega.$$

Here T_R^* is the real adjoint and the doubled space has its direct-sum real inner product. Then $\ker D_R$ has real dimension two and the nonzero singular gap is one. The extended operator $D_J = D_R \otimes 1$ is $\mathbb{C}_J$ -linear and self-adjoint, and its kernel

$$E_2 = \ker D_R \otimes_{\mathbb{R}} \mathbb{C}_J$$

has complex dimension two with the same nonzero gap.

Proof. In the orbit basis the real cut is the length-six truncated unilateral shift. Its singular values are five ones and one zero. Thus the doubled kernel is

$$\mathbb{R}(K^{-1}\Omega, 0) \oplus \mathbb{R}(0, K\Omega).$$

Tensor extension preserves these finite-matrix kernel and singular-value statements, while scalar multiplication commutes with $D_R \otimes 1$ by construction. $\square$

The in/out vectors belong to different doubled summands. Their microscopic labeling need not define a retained right polarization. The physical retention of E_2 and the scalar-extension rule remain explicit interface assumptions. The result repairs compatibility; it does not derive that physical interface from the source algebra alone.

Theorem A.19 (Finite spectral protection). *Unitary transport of the exact source pair preserves the chord response $\Delta_3 = 2\sin(\pi/7)$ and the scalar-extended doubled defect gap $\Delta_2 = 1$. The defect zero cluster is isolated by distance one. The positive-frequency cluster of $-i\log K$ has rank three and is separated from zero by $2\pi/7$. For this Hermitian generator and for D_J , self-adjoint perturbations smaller than half their respective cluster-separating distances preserve the corresponding ranks. More generally a Riesz rank persists when the resolvent remains invertible on a fixed enclosing contour.*

Proof. The clock eigenvalues are all seventh roots of unity, so the smallest nonzero chord is $2\sin(\pi/7)$ and the smallest positive logarithmic frequency is $2\pi/7$. The doubled truncated-shift calculation gives the defect gap one. Unitary conjugation preserves these spectra. For a self-adjoint matrix, a perturbation of norm η moves the spectrum by at most η ; a cluster and its complement therefore remain separated when 2η is below their initial separation. Equivalently, the contour formula for the spectral projector depends continuously on a resolvent that stays invertible there. Its integer rank is constant along the resulting continuous deformation. $\square$

Remark A.20. The chord response is not the positive-frequency Riesz separation: $|I - K|$ has conjugate-pair degeneracies and by itself does not select the complex triplet. Protection of a whole cluster also does not imply degeneracy of all internal eigenvalues. A frame change is covariant even when its matrix generator is nondegenerate; arbitrary $U(3)$ matrices need not commute with the fixed clock generator.

A.4 Sensitivity to interface choices

The finite source proof is conditional on associativity and a positive multiplicative norm; the quadrature count is conditional on the independent two-plane assignment; the doublet is conditional on the real doubled cut and subsequent scalar extension. None is a consequence of the common-source ontology alone. Source order, carrier rank, internal phase, defect gap, and physical spatial dimension are distinct quantities.

The standard-form channel uses these modules in commuting left and right representations. It does not require every microscopic selector to be absent from every possible prepared joint state. The determinant-linked group additionally requires retention of the product volume and absence of extra named right tensors. Removing that volume gives $U(3) \times U(2)$; retaining separate volumes instead gives $SU(3) \times SU(2)$. These alternatives share the same local rank and phase data.

B Amplitude reduction and memory are distinct from curvature

B.1 Exact reduction: what completion can hide

Let $C : \mathcal{H} \rightarrow \mathcal{H}_{\text{rec}}$ be a coisometry, $CC^* = I$, and write $P = C^*C$. Let $U(t) = e^{-itH/\hbar}$ for a finite-dimensional self-adjoint H . Define the compressed evolution $T(t) = CU(t)C^*$.

Theorem B.1 (Closure and the missing-record response). *The compressed family is a unitary group for all real t if and only if $[P, H] = 0$. In general,*

$$I - T(t)^*T(t) = CU(t)^*(I - P)U(t)C^*, \quad (10)$$

$$I - T(t)^*T(t) = \frac{t^2}{\hbar^2}CH(I - P)HC^* + O(t^3). \quad (11)$$

Proof. Use $CC^* = I$ and $C^*C = P$ to obtain the first identity. Its quadratic form is the squared norm of $(I - P)U(t)C^*$. If $T(t)$ is unitary for every t , that norm vanishes and $\text{Ran } P$ is invariant under both $U(t)$ and $U(-t)$, hence reducing. Differentiation gives $[P, H] = 0$. Conversely a reducing subspace carries the restricted unitary group. Expansion of the missing amplitude gives $(I - P)U(t)C^* = -it(I - P)HC^*/\hbar + O(t^2)$, which proves the second formula. $\square$

Compression of amplitudes is not automatically a trace-preserving quantum channel. An environment, noise, or a specified measurement instrument is needed when that interpretation is intended. In particular a nonunitary amplitude reduction is not by itself a particle-creation calculation.

Writing a linear evolution in retained and hidden components gives a self-contained memory formula. For constant bounded blocks,

$$\dot{x} = L_{PP}x + L_{PQ}y, \quad \dot{y} = L_{QP}x + L_{QQ}y,$$

variation of constants yields exactly

$$\dot{x}(t) = L_{PP}x(t) + L_{PQ}e^{tL_{QQ}}y(0) + \int_0^t L_{PQ}e^{(t-s)L_{QQ}}L_{QP}x(s) \, ds. \quad (12)$$

Thus a local enlarged evolution produces a nonlocal reduced equation. This is the elementary block form of projection-based memory methods [13, 14, 15] and connects to the dual-time representation of memory in [4]. Dissipation needs its own stability or bath assumptions; complete positivity is a separate condition [16, 17].

B.2 Three distinct obstructions

Temporal path dependence, retained-fiber curvature, and reduced memory are different quantities. For two supplied temporal connections,

$$\mathcal{F}_{st} = \partial_s \mathcal{A}_t - \partial_t \mathcal{A}_s + [\mathcal{A}_s, \mathcal{A}_t].$$

Vanishing $\mathcal{F}_{st}$ gives local flat transport on a simply connected patch; it does not lower a two-dimensional parameter manifold to one dimension. Nonzero curvature makes contractible loop transport depend on loop geometry, not just on ordinary homotopy class. To obtain a spacetime gauge field from temporal curvature one needs a physical map of record paths into this temporal construction and compatible retained representations. No such map follows just from the use of the word connection.

The memory kernel in (12) can be nonzero in a model with no spatial connection at all. Conversely a smoothly varying protected subspace can have gauge curvature under unitary adiabatic transport. On symplectic one-particle scattering spaces the antilinear part is $\beta = \frac{1}{2}(T + J_{\text{out}}TJ_{\text{in}})$; $\beta = 0$ is phase compatibility, not flatness. Particle-number interpretations additionally require a Fock representation and the usual implementability conditions. These distinctions prevent an irreversibility argument from being mistaken for a derivation of Yang–Mills curvature.

C Spatial dimension is independent of the finite source

The SMM/DTT interpretation seeks to obtain spatial multiplicity from inner generation and completed records. The seven-state recurrence here selects three positive-frequency modes and one reset state, but matching these counts to three spatial directions and an outer time does not provide a canonical displacement map: the reset state is not derived as a macroscopic time coordinate, and a spectral mode is not a direction of motion between records. The quaternionic source has a three-dimensional real imaginary module, and its reflection-odd clock output is a copy of that module. Neither statement identifies it with spatial displacements between completed records. The following independent counterexample makes the distinction precise.

Proposition C.1 (Spatial dimension is independent of the finite clock). *For each integer $q \geq 1$, the seven-state source, scalar-extended defect, standard-form record channel, and retained product-volume tensor admit a translation-invariant realization over a record graph with spatial volume growth of degree q and random-walk return exponent $q/2$. The comparisons may be flat in every realization. Consequently the finite source and declared local record operations do not select three spatial dimensions, even when the outer records are assigned a common chronological index.*

Proof. Take $G_q = (\mathbb{Z}^q, E_q)$ with its nearest-neighbor edges. At each vertex use an identical copy of the finite source, defect, and local channel from Appendices A.2 and A.3

and Section 4. Equip each retained carrier with the same total-volume tensor, and choose identity comparators along edges in a product trivialization. These are tensor compatible and have trivial holonomy; a change of independently chosen vertex frames gives the covariant links $g_x^{-1}g_y$. No vertex records a distinguished internal basis. The number of vertices within graph distance r is $|B_x(r)| = \Theta(r^q)$. For nearest-neighbor simple random walk, Fourier inversion gives

$$p_{2n}(x, x) = \frac{1}{(2\pi)^q} \int_{[-\pi, \pi]^q} \left(\frac{1}{q} \sum_{i=1}^q \cos k_i \right)^{2n} d^q k = \Theta(n^{-q/2}).$$

The exponents vary with q , while all finite source and channel identities remain the same. Copies indexed by an additional discrete outer time n preserve the same conclusion on each chronological slice; the index alone specifies neither a Lorentzian metric nor a causal light cone. $\square$

The counterexample does not say that q -dimensional graphs are generated by DTT. It shows that the present axioms leave their adjacency unspecified. If a microscopic update is proposed to form adjacency rather than assuming it, its dimension claim should pass the following independent tests:

- (D1) **Derived record graph.** Define completed events, a source-induced relation of causal influence or finite-range comparison, and its adjacency and edge weights using the microscopic update alone. Prove that the rule selects a robust large-scale graph class without stipulating a cubic lattice.
- (D2) **Spatial scaling.** For growing finite graphs of linear scale L and $\Theta(L^3)$ vertices, prove $|B_x(r)| \asymp r^3$ on expanding ranges $1 \ll r \ll L$, and for a specified unbiased diffusion prove a corresponding return law $p_{2n}(x, x) \asymp n^{-3/2}$ for $1 \ll n \ll L^2$, with uniform constants away from boundaries. The second exponent is the spatial spectral dimension diagnostic, not the dimension of the seven-state Hilbert space [7]. Neither scaling law by itself establishes a smooth three-manifold.
- (D3) **Source-to-displacement map.** Construct from the same update an operational displacement or coframe map into the reflection-odd source module $W_s \simeq \text{Im } \mathbb{H}$. Prove that its continuum limit has rank three, a nondegenerate metric induced by the quaternionic norm, and transition functions that identify source $SO(3)$ with rotations of *spatial tangent frames*. Without this map, the $SO(3)$ of Lemma A.14 remains an internal output isotropy. Verify in the joint record channel that the spatial source-frame action and the retained internal right-frame action remain independent: the coframe must not select a preferred mixed spatial–internal tensor that reduces the claimed gauge stabilizer.
- (D4) **Causal limit.** Derive chronological ordering, an appropriate local causal propagation rule, and a four-dimensional Lorentzian continuum with three-dimensional spatial slices. Establish this limit in addition to the two spatial scaling tests; a discrete clock or a separately attached time index does not imply a causal metric.

These are proof obligations, not results of the present finite construction. They also keep distinct the split-complex temporal symbol j , when used in DTT with $j^2 = +1$, and the selected ordinary complex phase J with $J^2 = -I$ in Appendix A.3. The powers of j

alternate between 1 and j ; without an additional dynamical representation they do not generate three spatial displacement directions. A bridge between these two structures must specify its action on completed-event correlations and establish the spatial and causal limits above.

C.1 An operational displacement criterion

To turn the source module into space, attach to a comparison history a displacement $\xi(h) \in W = \text{Im } \mathbb{H}$. In a flat spatial presentation require

$$\xi(h_2 h_1) = \xi(h_2) + \xi(h_1), \quad \xi(h^{-1}) = -\xi(h). \quad (13)$$

This is a candidate dynamical output, not a definition of spatial dimension. If it vanishes on closed spatial comparison histories, it integrates to $f(v) = \xi(h_{o \rightarrow v})$, independently of the path. Nonzero translational periods require local charts or a covering space. With varying spatial frames, the addition law uses the appropriate spatial parallel transport; the flat formula does not impose zero gauge curvature on the independent internal carriers.

Proposition C.2 (A sufficient three-dimensional growth criterion). *Suppose a connected bounded-degree record graph admits a map $f : \mathcal{V} \rightarrow W$ with uniformly bounded fibers, whose distinct image points are uniformly separated and form a relatively dense subset of W . Assume constants $a \geq 1, b \geq 0$ satisfy*

$$a^{-1} d_\Gamma(v, w) - b \leq |f(v) - f(w)| \leq a d_\Gamma(v, w) + b.$$

Then its graph balls obey $|B_v(r)| \asymp r^3$ for large r .

Proof. The right inequality places the image of a graph ball in a Euclidean ball of radius $ar + b$. Uniform separation bounds the number of image points there by a constant times r^3 , and bounded fibers give the same upper bound for vertices. Relative density supplies a constant times r^3 image points in a Euclidean ball of suitably smaller radius around $f(v)$. The left inequality puts preimages of those points inside the graph ball after decreasing that radius by a fixed factor and additive constant. This proves the lower bound. $\square$

The proposition explains why a source-derived map would matter: an operationally faithful, noncollapsed displacement geometry transfers the source's dimension to record growth. Merely writing a map into W is insufficient. Relative density, bounded fibers, and the two-sided distance control are substantial dynamical requirements. They exclude hidden macroscopic directions and collapsed or space-filling encodings.

Diffusion is a second test. In the explicit translation-invariant case, let a symmetric finite-range lazy walk on $\mathbb{Z}^3$ have step probabilities $p(s)$, with $p(0) > 0$, generating support, and positive covariance $\Sigma = \sum_s p(s) s s^T$. Its characteristic function is

$$\widehat{p}(k) = 1 - \frac{1}{2} k^T \Sigma k + O(|k|^4).$$

Fourier inversion, exponential suppression away from $k = 0$, and the change $k = n^{-1/2} q$ give

$$p_n(0, 0) \sim \frac{1}{(2\pi n)^{3/2} \sqrt{\det \Sigma}}. \quad (14)$$

This is a self-contained diffusion witness, not a claim that temporal generation has already produced its lattice or step distribution. On irregular networks one must prove appropriate

heat-kernel control; volume growth alone does not rule out bottlenecks and anomalous diffusion.

An additional smooth limit can turn ξ into a coframe $e : T\Sigma \rightarrow W$ of rank three. Then $h(X, Y) = \langle e(X), e(Y) \rangle_W$ is a spatial metric. The source $SO(3)$ acts on this coframe, while the independent right $S(U(3) \times U(2))$ action acts on internal fibers. Deriving this factorization is indispensable: equating spatial rotations with the weak gauge group would change the physical symmetry rather than explain it.

D Reproducibility and availability

The results are finite-dimensional identities or conditional transport statements with proofs supplied in the text. The accompanying verification scripts check the polar phase, marked cut, selector channel, auxiliary register, joint right-algebra retention, and reference transfer. The new completion verifier tests 35 reservoir density states, the coherence bound and saturating transfer, generic two- and three-dimensional probe fixed algebras with selector/right dimensions 2/2, the six-dimensional label algebra, all 16 right matrix units for two rank-two carriers, and a retained Bell correlation. These are floating-point finite-model checks. Numerical checks support implementation and do not replace the proofs or establish the physical premises.

Data and code availability. The source, verification code, and calculated outputs are supplied as accompanying material. All numerical inputs are explicit finite matrices and dimensionless parameters; no experimental dataset is used.

Declarations

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