

Gauge Fields from Temporal Connections: A Non-Commutative Approach

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Abstract

We propose a non-commutative geometric formulation of gauge theory grounded in the temporal structure of the Duality of Time Theory (DTT). In this framework, gauge fields emerge as connections over the discrete inner-time dimension. We define a spectral triple over dual-time space, and demonstrate how Yang–Mills fields arise as fluctuations of the Dirac operator along inner-time cycles. This leads to a novel interpretation of gauge interactions as modulations of temporal coherence.

1 Introduction

Gauge theories provide the backbone of modern particle physics, yet their geometric interpretation remains rooted in spatial fiber bundles. In contrast, the Duality of Time Theory (DTT) postulates that space itself is emergent from an inner temporal dynamic. Here, we investigate the possibility that gauge fields are not spatial connections but arise from generalized temporal derivatives on a non-commutative temporal manifold.

2 Temporal Spectral Triple

We define a spectral triple $(\mathcal{A}, \mathcal{H}, D)$ over a dual-time geometry:

- $\mathcal{A}$: algebra of functions on discrete inner-time cycles;
- $\mathcal{H}$: Hilbert space of spinor-valued fields on dual-time;
- D : Dirac operator incorporating inner-time differentials.

Let τ denote the discrete inner-time coordinate and T the outer time. The Dirac operator takes the form:

$$D = i\gamma^0 \partial_T + i \sum_{j=1}^3 \gamma^j \nabla_j + i\gamma^5 \nabla_\tau, \quad (1)$$

where ∇_τ is the inner-time covariant derivative.

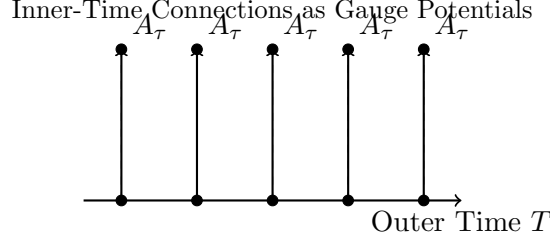


Figure 1: Temporal connections A_τ linking discrete inner-time re-creation events along the outer-time axis.

3 Gauge Fields as Temporal Connections

In non-commutative geometry, gauge fields emerge as inner fluctuations of the Dirac operator:

$$D \rightarrow D_A = D + A + JAJ^{-1}, \quad (2)$$

where A is a self-adjoint one-form in the differential algebra of $\mathcal{A}$, and J is the real structure.

We define the gauge potential A_τ as a one-form over inner-time:

$$A_\tau = \sum a_i[D, b_i], \quad a_i, b_i \in \mathcal{A}, \quad (3)$$

so that the temporal covariant derivative becomes:

$$\nabla_\tau = \partial_\tau + A_\tau. \quad (4)$$

This formulation implies that gauge interactions originate from modulations of coherence between successive inner-time re-creation events.

4 Comparison with Conventional Gauge Theories

Feature	Conventional Gauge Theory	DTT Temporal Gauge Theory
Base Space	Spatial manifold	Dual-time (discrete τ)
Connection	Fiber bundle connection	Temporal derivative operator
Gauge Field	$A_\mu(x)$	$A_\tau(\tau)$
Curvature	$F_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$	$F_{\tau T} = [\nabla_\tau, \partial_T]$
Interpretation	Local symmetry of fields	Temporal coherence modulation
Spectral Origin	Postulated Lagrangian	Derived from Dirac fluctuation

Table 1: Comparison of standard gauge theory and DTT-based temporal gauge construction.

5 Spectral Action and Yang–Mills Terms

The spectral action is given by:

$$S = \text{Tr} \left(f \left(\frac{D_A^2}{\Lambda^2} \right) \right), \quad (5)$$

where f is a cut-off function and Λ is a scale parameter. Expansion yields:

$$S = \int (R + \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \dots), \quad (6)$$

where $F_{\tau T}$ and other components arise from commutators of the inner-time connection:

$$F_{\tau T} = [\nabla_\tau, \partial_T]. \quad (7)$$

Thus, standard Yang–Mills terms are encoded in the inner-time geometry, and coupling constants are linked to temporal scales.

6 Example: $U(1)$ Temporal Gauge Field

Consider a simple $U(1)$ temporal gauge connection:

$$A_\tau = ia(\tau), \quad a(\tau) \in \mathbb{R}. \quad (8)$$

The curvature becomes:

$$F_{\tau T} = i\partial_T a(\tau), \quad (9)$$

indicating an electric-type field originating from inner-time modulations.

7 Discussion and Future Work

This framework reinterprets gauge fields as manifestations of internal time coherence, rather than fiber bundle connections over space. Further directions include:

- Extending to non-Abelian groups ($SU(2)$, $SU(3)$) via matrix-valued inner-time algebras.
- Exploring spontaneous symmetry breaking as a temporal phase transition.
- Connecting with observable quantum phase effects and anomalies.

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References